Class	Full Name	Index Number

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PRELIMINARY EXAMINATION 2010

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4016/ 02

MATHEMATICS
Paper 2
Secondary 4 Express/ 5 Normal Academic/ 4A1

14th September 2010 **2 hours 30 minutes**

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the writing paper.

Answer all the questions.

Write your answers and working on the writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

At the end of the examination, fasten all your work securely together.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's Use

This document consists of **11** printed pages (including this cover page).

Setter: Ms J Toh

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Evaluate $\dfrac{9.85\times 10^5 \times \pi - 25.8^3}{2.6^8 + \sqrt{8200}}$ giving your answer in standard form. [1]

(b) Given $64^{y+1} = 8^{2y-1} \times 4^{2y+3}$, find the value of y. [2]

(c) There are 9 three-room units in each level of a HDB block. A painter can finish painting the walls of 1 level of three-room units in 2 days. How many painters are needed to finish painting the walls of all the three-room units in a HDB block with 12 levels in 6 days? [2]

(d) A family of 8 adults and 6 children took a bus trip to Kuala Lumpur. The bus left Singapore for Kuala Lumpur at 2015 on Wednesday.

(i) If the bus arrives in Kuala Lumpur at 0610 on Thursday, find the time taken for the bus journey, in hours and minutes. [1]

(ii) If the fare of an adult ticket and a child ticket from Kuala Lumpur back to Singapore is RM24.85 and RM15.50 respectively, calculate the total amount of fares paid for the above trip in S$, given that the exchange rate is S$100 = RM228.50. [2]

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2 (a) Factorise completely $10x^2 + xy - 3y^2$. [2]

(b) Express $\dfrac{9 - x^2}{3x^2 - 8x - 3} + \dfrac{9}{3 - x}$ as a single fraction in its simplest form. [3]

(c) Solve the equation $3(2x^2 - 9) = 48$, giving your answers correct to 2 decimal places. [2]

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3 (a) Danny saved 13% of his monthly income and spent 20% of the remainder on transport.

(i) If Danny spends \$487.20 on transport, calculate his monthly income. [2]

(ii) If Danny deposits 10 months of his monthly savings into a bank which pays a simple interest of 0.25% per annum, find the amount of money in the bank after 20 months. [2]

(b) Danny bought a microwave oven at a discounted price of \$864 during the Great Singapore Sales.

(i) If the shopkeeper has given Danny a discount of 28%, find the selling price of the microwave oven before the discount. [2]

(ii) If the shopkeeper still makes a profit of 5% after giving Danny a 28% discount, find the cost price of the microwave oven. [2]

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4 The equation of a straight line AB is $\frac{x}{2}+\frac{y}{2}=1$.

(a) Find the gradient of line AB. [1]

(b) Find the y - intercept of line AB. [1]

(c) Find the coordinates of the point at which line AB intersects the line $y = 2$. [1]

(d) A second line PQ is parallel to the line AB and passes through a point $(2,-8)$.
Show that the second line PQ also passes through the point (-1, -5). [2]

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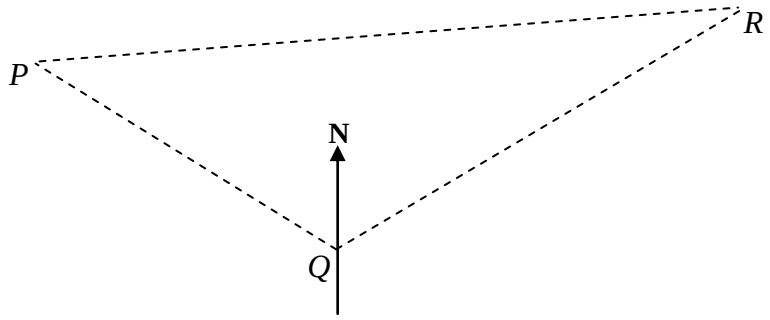
5 (a) A ship sails from P to Q 15 km away. It then sails 18 km from Q to R on a bearing of $074°$.

Given that $\angle PQR = 142°$, calculate



(i) the bearing of Q from P, [1]

(ii) how far is Q due east of P, [2]

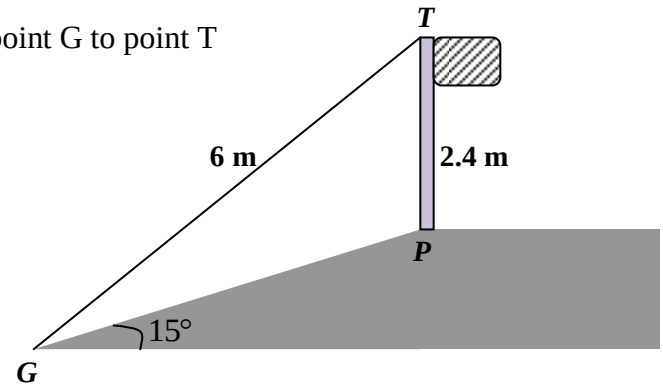
(iii) the distance PR, [3]

(iv) the shortest distance taken by the ship to sail from R to a position T, which is due north of Q. [2]

(b) A flag pole of 2.4 m stands at a point P on the top of a slope which is inclined at $15°$ to the horizontal gound. A boy ties a 6 m taut rope to the top of the flag pole at point T and attached it to a point G at the end of the slope. Calculate

(i) $\angle TPG$ [1]

(ii) the angle of elevation from point G to point T [3]



6 A bee is flying to its destination 6 km away. The speed of the bee in still air is 2.4 km/h and the speed of the wind, which is constant throughout, is x km/h.

(a) Write down expressions in terms of x, for the time taken by the bee, in hours, if it is flying

(i) against the wind, [1]

(ii) in the same direction as the wind. [1]

(b) The time taken to fly against the wind is 30 minutes longer than the time taken to fly in the same direction as the wind.
Write down an equation in terms of x and show that it reduces to

$$x^2 + 24x - 5.76 = 0 .$$ [3]

(c) Solve the equation to find the speed of the wind. [3]

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7 Three men, Ace, Ben and Con were assigned on a mission to rescue survivors from a landslide. They needed to travel along a river by boat in order to reach the disaster area. The three men needed to carry individual loads of food and medical supplies with them. The weights of the three men and the supplies they were carrying are shown in the table below.

Name	**Personal Mass (kg)**	**Mass of loads (kg)**
Ace	75	20
Ben	62	18
Con	65	15

(a) Write down a 3×2 matrix $\boldsymbol{R}$ to represent the masses of the three men. [1]

(b) If the men were charged \$0.12 per kg of personal mass and \$0.80 per kg of load mass for the boat ride,

(i) write down a 2×1 matrix $\boldsymbol{P}$ to represent the costs per kg. [1]

(ii) evaluate $\boldsymbol{RP}$ and explain what the elements of $\boldsymbol{RP}$ represent. [2]

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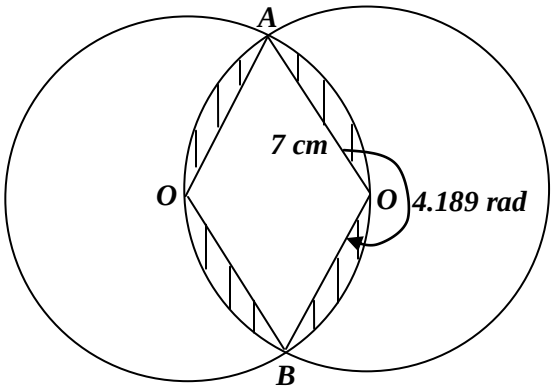
8 (a) The diagram below shows two circles intersecting at ***A*** and ***B***. The circles have centres ***O*** and radii 7 cm.

Given reflex $\angle AOB = 4.189$ *rad*, find the area of the shaded region. [3]



(b) Fig 1 shows a hemispherical container with a capacity of 1500 cm^3.

Fig 2 shows a hexagonal container with equal sides of 5 cm and a height of 9 cm.

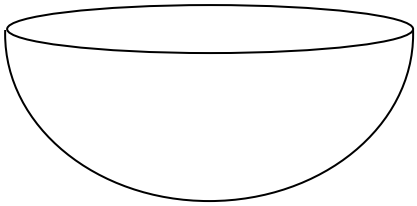


Fig 1

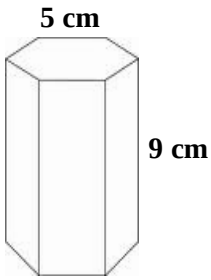
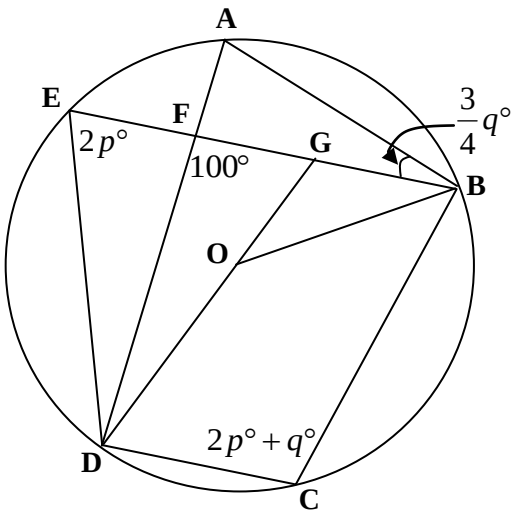


Fig 2

(i) Find the radius of the hemispherical container. [2]

(ii) Water is filled to the brim in the hexagonal container.
What is the minimum number of times it will take to transfer enough water to fill the hemispherical container? [3]

9 In the diagram, A, B, C, D and E are points on the circumference of a circle with centre O such that GO = GB, $\angle DFB = 100°$, $\angle DEB = 2p°$, $\angle ABE = \frac{3}{4}q°$ and $\angle DCB = 2p° + q°$.



(a) Explain clearly why

(i) $2p° + 2p° + q° = 180°$ [1]

(ii) $2p° + \frac{3}{4}q° = 100°$ [2]

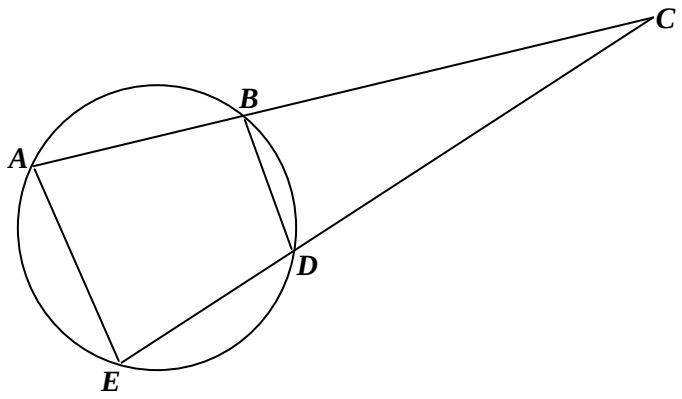
(b) Find the values of $p°$ and $q°$. [2]

(c) Hence, find

(i) reflex $\angle DOB$ [2]

(ii) $\angle DBG$ [2]

10 The diagram shows a cyclic quadrilateral ABDE whereby AB and ED are extended to meet at point C. Given that BD = 6 cm and AE = 9 cm,



(a) Explain clearly why ΔCBD is similar to ΔCEA. [2]

(b) Find the area of ABDE if the area of ΔBCD = 30 cm^2. [2]

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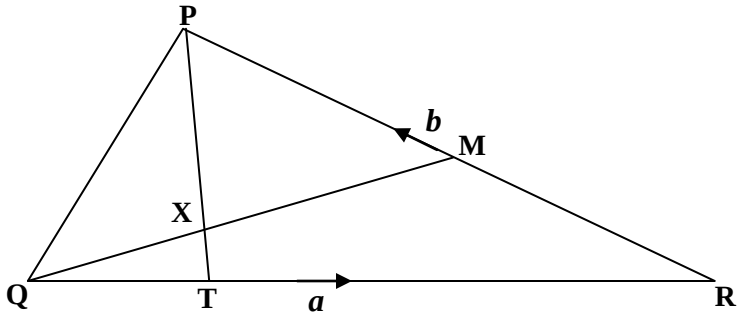
11 M is the midpoint of PR, $QR = 4QT$ and $TP = 5TX$. Given that $\overrightarrow{QR} = \boldsymbol{a}$ and $\overrightarrow{RP} = \boldsymbol{b}$,



(a) express, as simply as possible, in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$,

(i) $\overrightarrow{QT}$, (ii) $\overrightarrow{QM}$, (iii) $\overrightarrow{TP}$. [3]

(b) show that $\overrightarrow{QX} = \frac{2}{5}(\boldsymbol{a} + \frac{1}{2}\boldsymbol{b})$. [2]

(c) calculate the value of

(i) $\frac{QX}{QM}$, [1]

(ii) $\frac{area\Delta PQX}{area\Delta PQM}$, [1]

(iii) $\frac{area\Delta PQX}{area\Delta PQR}$. [1]

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12 (a) The table below shows the masses of 200 newborn babies in Hospital A.

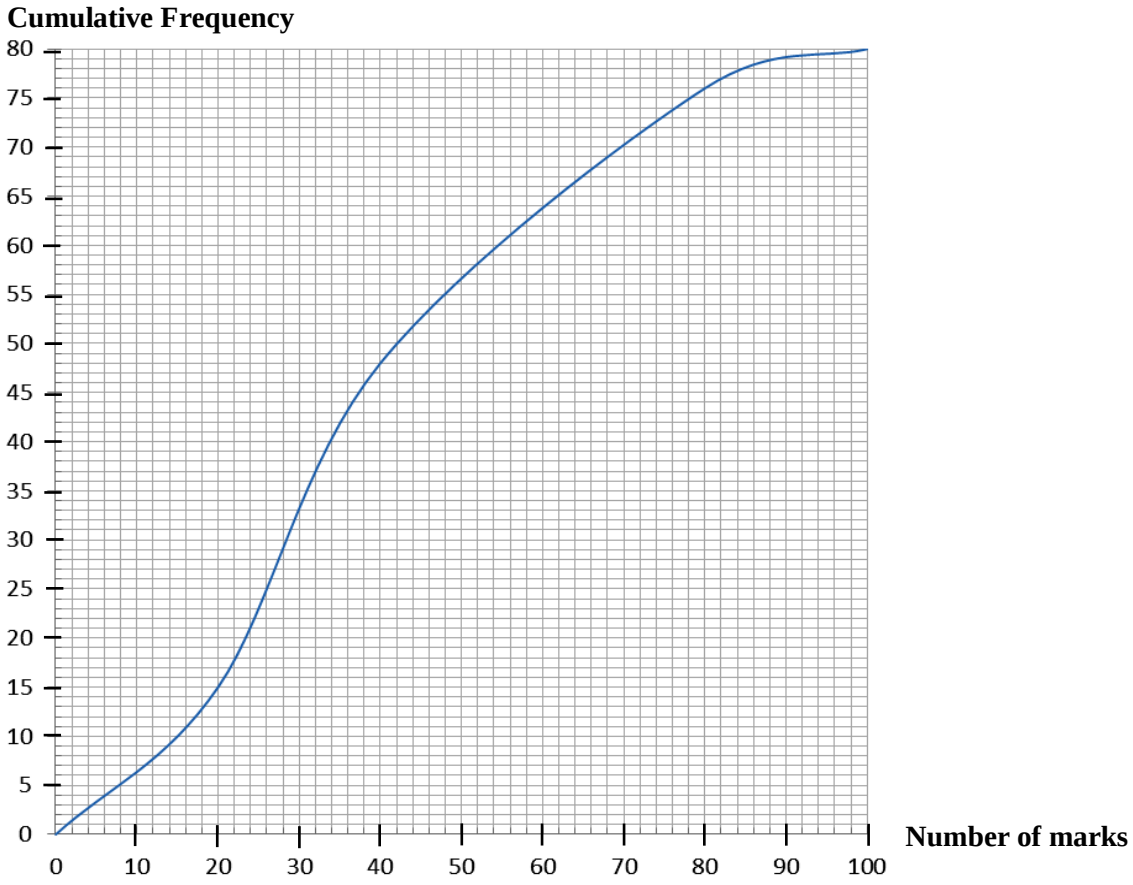
Mass (x kg)	$1.5 < x \leq 2.0$	$2.0 < x \leq 2.5$	$2.5 < x \leq 3.0$	$3.0 < x \leq 3.5$	$3.5 < x \leq 4.0$
Number of babies	10	25	60	80	25

(i) Calculate the mean mass in Hospital A. [2]

(ii) Calculate the standard deviation in Hospital A. [2]

(iii) Given the mean mass in Hospital B = 2.795 kg and standard deviation in Hospital B = 0.748, compare the masses of babies born in the two hospitals. [1]

(b) 80 pupils took part in an inter-school Mathematics Quiz. The cumulative frequency curve below shows the distribution of the quiz results.



(i) Find the interquartile range [2]

(ii) Given that the top 6.25% of the pupils are awarded a certificate of distinction in the quiz, and if 2 pupils are chosen at random, find the probability that both pupils obtained a certificate of distinction. [2]

13 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = 2\left(\frac{3}{x} + 2x - 5\right)$.

Some of the corresponding values are given in the following table.

x	0.5	1	1.5	2	2.5	3	3.5	4
y	4	0	***p***	1	2.4	4	***q***	7.5

(a) Calculate the value of ***p*** and ***q***. [1]

(b) Using 4 cm to represent 1 unit, draw the horizontal axis for $0 \leq x \leq 4$ and 2 cm to represent 1 unit, draw the vertical axis for $-1 \leq y \leq 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [2]

(c) Use your graph to

(i) find the smallest value of $2\left(\frac{3}{x} + 2x - 5\right)$ in the interval $0 \leq x \leq 4$. [1]

(ii) find the gradient of the curve at $\boldsymbol{x} = 2$ by drawing a tangent. [2]

(iii) solve the equation $\frac{3}{x} + 2x - 5 = 2$. [2]

(d) The line $y = -2x + a$ intersects the curve $y = 2\left(\frac{3}{x} + 2x - 5\right)$ at $x = 0.5$ and $x = \boldsymbol{b}$.

Draw the line $y = -2x + a$ on the same axes and find the values of a and b. [2]

Answers

1(a) 1.41×10^{3} (b) $\frac{3}{4}$ (c) 4 (d)(i) 9hrs 55min (ii) S$127.70

2(a) $(5x + 3y)(2x - y)$ (b) $\frac{x(x+27)}{(3x+1)(3-x)}$ (c) 3.54 or -3.54

3(a)(i) $2800 (ii) $3655.17 (b)(i) $1200 (ii) $822.86

4(a) -1 (b) 2 (c) (0, 2)

5(a)(i) 112° (ii) 13.9 (iii) 31.2 (iv) 17.3
5(b)(i) 105° (ii) 37.7°

6(a)(i) $\frac{6}{2.4-x}$ (ii) $\frac{6}{2.4+x}$ (c) 0.238

7(a) $\begin{pmatrix}75 & 20\\ 62 & 18\\ 65 & 15\end{pmatrix}$ (b)(i) $\begin{pmatrix}0.12\\ 0.8\end{pmatrix}$ (ii) $\begin{pmatrix}25\\ 21.84\\ 19.8\end{pmatrix}$

7(d) The elements represent the total individual cost for the boat ride of Ace, Ben and Con respectively.

8(a) 17.7 (b)(i) 8.95 (ii) 3

9(a)(i) angles in opposite segments (ii) angles in the same segment and exterior angles of Δ
9(b)(i) 220° (ii) 60°

10(a) AAA (b) 37.5

11(a)(i) $\frac{1}{4}\mathbf{a}$ (ii) $\mathbf{a} + \frac{1}{2}\mathbf{b}$ (iii) $\frac{3}{4}\mathbf{a} + \mathbf{b}$

11(c)(i) $\frac{2}{5}$ (ii) $\frac{2}{5}$ (iii) $\frac{1}{5}$

12(a)(i) 2.9625 (ii) 0.511
12(iii) Average mass of babies in Hosp A is heavier as indicated by the larger mean;
Less variation in the spread of masses in Hosp A as indicated by the smaller s.d.

12(b)(i) 31 (ii) $\frac{1}{316}$

13(a) p = 0 and q = 5.71 (c)(i) -0.2 (ii) 2.67 (iii) 0.5 or 3
13(d) a = 5 and b = 2